

Math 55 Discussion problems 7 Mar

1. How many ways are there for three penguins and six puffins to stand in a line so that
 - (a) all puffins stand together?
 - (b) all penguins stand together?
2. A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?
3. How many bit strings contain exactly five 0s and 14 1s if every 0 must be immediately followed by two 1s?
4. Give a combinatorial proof that

$$\sum_{k=1}^n k \binom{n}{k}^2 = n \binom{2n-1}{n-1}$$

[Hint: Count in two ways the number of ways to select a committee, with n members from a group of n mathematics professors and n computer science professors, such that the chairperson of the committee is a mathematics professor.]

5. Give a combinatorial proof that if n is a positive integer then

$$\sum_{k=0}^n k^2 \binom{n}{k} = n(n+1)2^{n-2}$$

[Hint: Show that both sides count the ways to select a subset of a set of n elements together with two not necessarily distinct elements from this subset. Furthermore, express the right hand side as $n(n-1)2^{n-2} + n2^{n-1}$.]

6. How many different combinations of pennies, nickels, dimes, quarters, and half dollars can a piggy bank contain if it has 20 coins in it?